

STEM@Home: Bending the Number Line

Can you imagine a world where the rules of math as you learned them don't apply? What if 2 plus 2 did not equal 4?

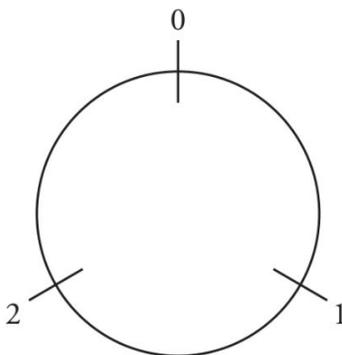
Can you believe that there is an entire branch of math, called modular arithmetic, that describes a completely different way (from the way you learn in school) of adding and subtracting—and that when you do modular arithmetic, you often get completely different answers? (What a great way to confuse your brother or sister.) In some instances of modular arithmetic, 2 plus 2 does not equal 4! We could have $2 + 2 = 1$, or even $2 + 2 = 0$!

Let's Get Started!

Before we continue, consider the number line. If you were to figure out $2 + 2$ on the number line, you would start with the number 2 and move two spaces to the right—this would take you to 4, so $2 + 2 = 4$. What would happen, though, if the number line weren't a line? We'll explore exactly this scenario.

To do modular arithmetic, you must first pick a number (the fancy term for this number is the modulus). The example that follows shows you how 2 plus 2 might not equal 4. We'll pick the number 3 as the modulus.

1. Draw a circle.
2. Draw three evenly spaced notches on the circle (the modulus you pick determines the number of notches you draw on the circle). Label the notches 0, 1, and 2, like so:



3. Counting and adding in modular arithmetic are the same thing as moving around the circle. Counting upward starting at 0, we'd have 0, 1, 2, 0, 1, 2, and so on.
4. Start at 2.
5. Now add 2 by moving from 2 to 0 to 1.
6. We added 2 to 2 and got 1 instead of 4!

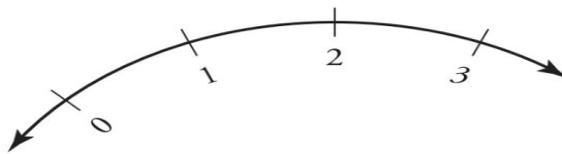
STEM@Home: Bending the Number Line (Cont.)

There actually is no 4 in this number system! A mathematician might say that $1 = 4 \pmod{3}$. Can you figure out why?

You can also do modular subtraction. What do you think $1 - 2$ might be in our example?

What other numbers would you like to try?

To think about how this works, imagine the number line. If we make it into a loop, and have the 3 meet the 0, we get the circle we just created!



As you might guess, the way that you add and subtract clock times is like modular arithmetic, using the numbers 60 (for minutes and seconds) and 12 (for hours) instead of 3. There are several other examples of ways to replace the rules of math to go to strange, exotic places.

For instance:

- Some types of arithmetic have addition but no multiplication, or multiplication but no division.
- In the regular arithmetic you know, A plus B always equals B plus A ; this (the commutative property) is not true in other kinds of arithmetic.
- There are kinds of geometry with no straight lines!