

STEM@Home: Math, Logically

You may remember learning that division by the number 0 is undefined—that's because there is no answer that fits the rules of arithmetic. (A quick refresher: Division is the inverse, or opposite, of multiplication. $10 \div 2 = 5$ because $5 \times 2 = 10$. You can't multiply any number by 0 to make 10, so the answer to the corresponding division problem is undefined.) This principle also explains why the square root of 4 is +2, even though $(-2)^2$ is also 4; mathematicians need standard rules and definitions.

Mathematicians are often very interested in finding where the rules break, or in coming up with new sets of rules that also work. (Modular arithmetic, from the "Bending the Number Line" STEM@Home sheet, is one.) Often, seeing where the rules break leads to new rules. The square root of 9 is 3, but -9 doesn't have a square root in the real numbers most of us use. And so the real numbers were extended to include the imaginary and complex numbers, where -9 does have a square root: $3i$.

To see where rules break, mathematicians start with basic rules, use logic to find more and more new rules, and see if any of those rules contradict each other. We won't get into mathematical logic and finding contradictions here; what we will do is explore contradiction in the rules of imaginary worlds.

What Is Logic?

Logic can be defined informally as a way of reasoning that makes sense. Here, we'll explore logical reasoning and illogical reasoning through a classic example: syllogisms.

A **syllogism** is a way of combining two statements that are true to create a third statement that is also true. For example:

All humans must breathe to live.

You are a human.

Therefore, you must breathe to live.

Makes sense, right? This is definitely logical. But sometimes people don't combine statements correctly. Think about this example:

All humans must breathe to live.

You must breathe to live.

Therefore, you are a human.

This time, the third statement is not true! You might actually be an alien, a puppy, or a redwood tree, as they all have to breathe, too.